In[2]:=
```
(* net revenue of a month (expanded Equation 2 in main text) *)
netMonth = c * d * a - dac * d * b - rac * (us * r - (us - uh) * d * a * h[n] / r)
```

Out[2]=
$$a\,c\,d - b\,d\,dac - rac\left(r\,us - \frac{a\,d\,(-uh + us)\,h[n]}{r}\right)$$

```
(* deriving the solution of the recursion
 Equation 15 (Equation 16 in main text) *)
happyRecursion = RSolve[{h[n] == us * r + (1 - us) * d * a * h[n - 1] / r, h[1] == r}, h[n], n]
```

Out[3]=
$$\left\{\left\{h[n] \to \frac{r^2\left(-a\,d\left(-\frac{a\,d\,(-1+us)}{r}\right)^n + r\left(-\frac{a\,d\,(-1+us)}{r}\right)^n + a\,d\,us\right)}{a\,d\,(-a\,d + r + a\,d\,us)}\right\}\right\}$$

```
(* getting the net revenue of each
  month from 1 to 12 with Equations 2 and 16) *)
netMonths = Table[Evaluate[netMonth /. First[happyRecursion]], {n, 1, 12, 1}] //
  FullSimplify
```

Out[4]=

$$\Big\{-b\,d\,dac - r\,rac\,us + a\,d\,(c + rac\,(-uh + us)),$$

$$-b\,d\,dac + \frac{a^2\,d^2\,rac\,(uh - us)\,(-1 + us)}{r} - r\,rac\,us + a\,d\,(c + rac\,us\,(-uh + us)),$$

$$a\,c\,d - b\,d\,dac - r\,rac\,us - \frac{1}{r^2}a\,d\,rac\,(uh - us)\,\left(a^2\,d^2\,(-1 + us)^2 + r^2\,us - a\,d\,r\,(-1 + us)\,us\right),$$

$$a\,c\,d - b\,d\,dac - r\,rac\,us + \frac{r\,rac\,(-uh + us)\left(-\frac{a^5\,d^5\,(-1+us)^4}{r^4} + \frac{a^4\,d^4\,(-1+us)^4}{r^3} + a\,d\,us\right)}{r + a\,d\,(-1 + us)},$$

$$a\,c\,d - b\,d\,dac - r\,rac\,us + \frac{r\,rac\,(-uh + us)\left(\frac{a^6\,d^6\,(-1+us)^5}{r^5} - \frac{a^5\,d^5\,(-1+us)^5}{r^4} + a\,d\,us\right)}{r + a\,d\,(-1 + us)},$$

$$a\,c\,d - b\,d\,dac - r\,rac\,us + \frac{r\,rac\,(-uh + us)\left(-\frac{a^7\,d^7\,(-1+us)^6}{r^6} + \frac{a^6\,d^6\,(-1+us)^6}{r^5} + a\,d\,us\right)}{r + a\,d\,(-1 + us)},$$

$$a\,c\,d - b\,d\,dac - r\,rac\,us + \frac{r\,rac\,(-uh + us)\left(\frac{a^8\,d^8\,(-1+us)^7}{r^7} - \frac{a^7\,d^7\,(-1+us)^7}{r^6} + a\,d\,us\right)}{r + a\,d\,(-1 + us)},$$

$$a\,c\,d - b\,d\,dac - r\,rac\,us + \frac{r\,rac\,(-uh + us)\left(-\frac{a^9\,d^9\,(-1+us)^8}{r^8} + \frac{a^8\,d^8\,(-1+us)^8}{r^7} + a\,d\,us\right)}{r + a\,d\,(-1 + us)},$$

$$a\,c\,d - b\,d\,dac - r\,rac\,us + \frac{a\,d\,r\,rac\,(-uh + us)\left(\frac{a^8\,d^8\,(a\,d-r)\,(-1+us)^9}{r^9} + us\right)}{r + a\,d\,(-1 + us)},$$

$$a\,c\,d - b\,d\,dac - r\,rac\,us + \frac{a\,d\,r\,rac\,(-uh + us)\left(-\frac{a^9\,d^9\,(a\,d-r)\,(-1+us)^{10}}{r^{10}} + us\right)}{r + a\,d\,(-1 + us)},\ a\,c\,d - b\,d\,dac -$$

$$r\,rac\,us + \left(r\,rac\,(-uh + us)\left(\frac{a^{12}\,d^{12}\,(-1 + us)^{11}}{r^{11}} - \frac{a^{11}\,d^{11}\,(-1 + us)^{11}}{r^{10}} + a\,d\,us\right)\right)\Big/$$

$$(r + a\,d\,(-1 + us)),\ a\,c\,d - b\,d\,dac - r\,rac\,us +$$

$$\left(r\,rac\,(-uh + us)\left(-\frac{a^{13}\,d^{13}\,(-1 + us)^{12}}{r^{12}} + \frac{a^{12}\,d^{12}\,(-1 + us)^{12}}{r^{11}} + a\,d\,us\right)\right)\Big/\ (r + a\,d\,(-1 + us))\Big\}$$

```
(* total net revenue for variable driver cost,
by summing up the net revenues (Equation 1 in main text) *)
totalNet = Total[netMonths] // FullSimplify // Cancel // FullSimplify
```

Out[5]=

$$\frac{1}{r^{11}}\left(a^{12} d^{12} \text{ rac } (uh - us) (-1 + us)^{11} - a^{11} d^{11} r \text{ rac } (uh - us) (-1 + us)^{10} (1 + us) + a^{10} d^{10} r^2 \text{ rac } (uh - us) (-1 + us)^9 (1 + 2\,us) - a^9 d^9 r^3 \text{ rac } (uh - us) (-1 + us)^8 (1 + 3\,us) + a^8 d^8 r^4 \text{ rac } (uh - us) (-1 + us)^7 (1 + 4\,us) - a^7 d^7 r^5 \text{ rac } (uh - us) (-1 + us)^6 (1 + 5\,us) + a^6 d^6 r^6 \text{ rac } (uh - us) (-1 + us)^5 (1 + 6\,us) - a^5 d^5 r^7 \text{ rac } (uh - us) (-1 + us)^4 (1 + 7\,us) + a^4 d^4 r^8 \text{ rac } (uh - us) (-1 + us)^3 (1 + 8\,us) - a^3 d^3 r^9 \text{ rac } (uh - us) (-1 + us)^2 (1 + 9\,us) + a^2 d^2 r^{10} \text{ rac } (uh - us) (-1 + us) (1 + 10\,us) - 12\, r^{11} (b\, d\, dac + r\, rac\, us) + a\, d\, r^{11} (12\, c - rac\, (uh - us) (1 + 11\,us))\right)$$

```
(* the derived threshold of driver cost (Equation 5 in main text) *)
driverCostThres = 2.33
```

Out[6]= 2.33

```
(* the linear relation for the number of rides
 offered by each driver (Equation 4 in main text) *)
aPiece = Piecewise[{{δ * c + ε, c ≥ driverCostThres}, {r / d, c < driverCostThres}}]
```

Out[8]=

$$\begin{cases} c\,\delta + \varepsilon & c \geq 2.33 \\ \frac{r}{d} & c < 2.33 \\ 0 & \text{True} \end{cases}$$

In[9]:=

```
(* the linear relation for the driver quit rate (Equation 7 in main text) *)
bPiece = (ξ * c + η)
```

Out[9]= $c\,\xi + \eta$

In[10]:=

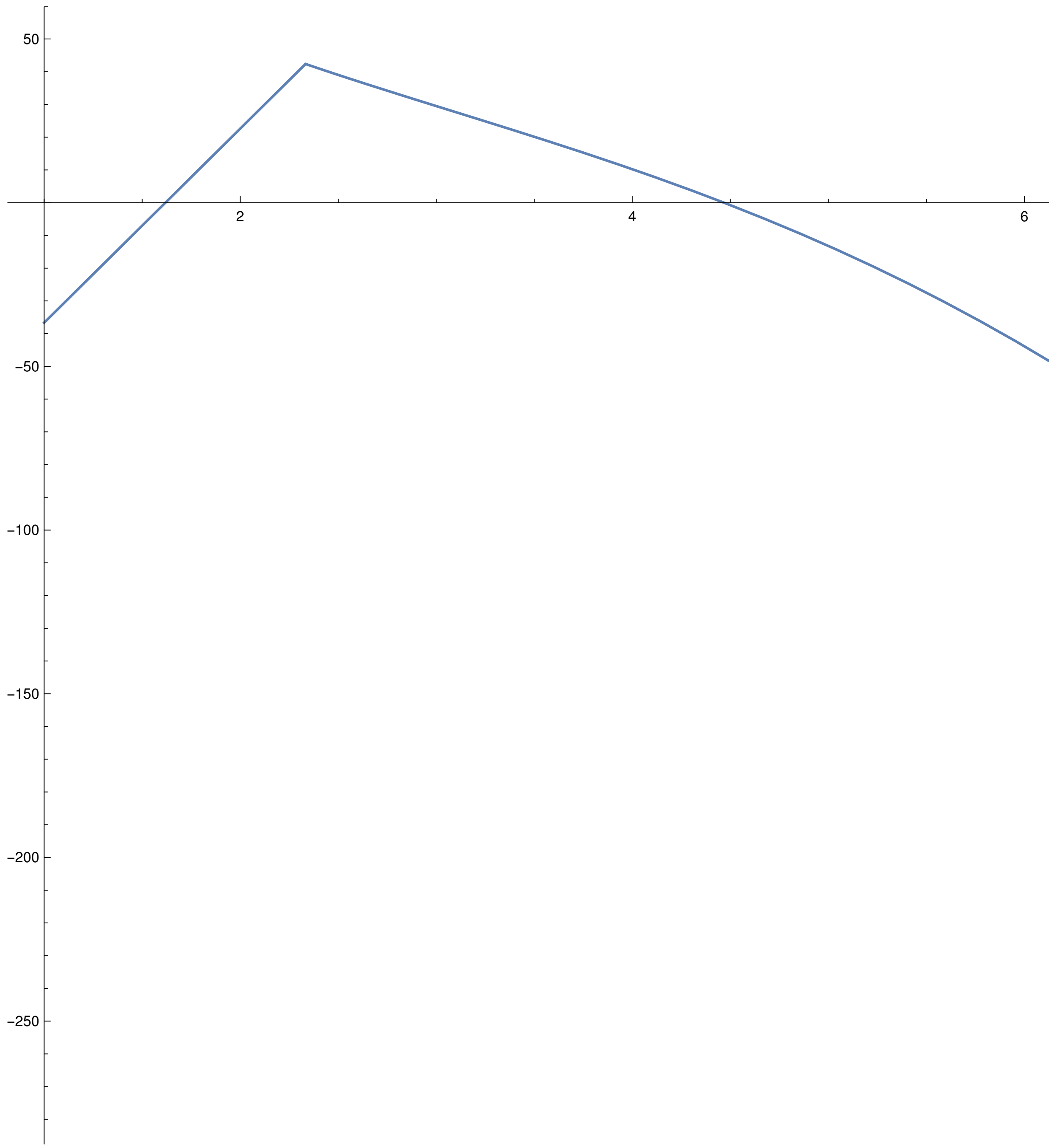
```
(* plugging the previous linear relations in the total net revenue relation *)
totalNetApiece = totalNet /. a → aPiece;
totalNetBpiece = totalNetApiece /. b → bPiece;
totalNetFun = (totalNetBpiece /. c → #) &;
```

```
(* the initial CAC for standardising the revenue *)
initCAC = 16 600 * 15 + 100 * 500
```

Out[∘]= 299 000

```
(* the plot of the 12-
 month total net revenue (y-axis) for different driver costs (x-axis) *)
Plot[Table[totalNetFun[c] / initCAC * 100, {dac, {500}},
  {rac, {15}}, {r, {16 600}}, {d, {100}}, {uh, {0.1}}, {us, {0.33}},
  {δ, {-18}}, {ε, {208}}, {ξ, {0.03667}}, {η, {-0.02}}], {c, 1, 10}]
```

Out[ ]=



In[13]:=

```
(* saving the plot's curve data to file,
for plotting it in R together with the simulation model's *)
totalNetPlotData =
  Cases[Plot[Table[totalNetFun[c] / initCAC * 100, {dac, {500}}, {rac, {15}},
     {r, {16 600}}, {d, {100}}, {uh, {0.1}}, {us, {0.33}}, {δ, {-18}}, {ε, {208}},
     {ξ, {0.03667}}, {η, {-0.02}}], {c, 1, 10}], Line@x__ → x, Infinity];
Export["/home/danis/Desktop/Amanda/Lyft/code/totalNetPlotData.txt",
 Partition[Flatten[totalNetPlotData], 2], "Table"]
"/home/danis/Desktop/Amanda/Lyft/code/totalNetPlotData.txt"
```

Out[14]= /home/danis/Desktop/Amanda/Lyft/code/totalNetPlotData.txt

Out[15]= /home/danis/Desktop/Amanda/Lyft/code/totalNetPlotData.txt